Enhancement of the Adjoint Method by Error Control of Accelerations for Parameter Identification in Multibody Dynamics

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ABSTRACT:
The potential of the adjoint method for a variety of optimization problems in multibody dynamics such as inverse dynamics and parameter identification is the focus of the present paper. Despite the complicated structure of the equations and matrices for the adjoint system, the additional effort when combining the standard forward solver to the adjoint backward solver, is kept in limits. Hence, the adjoint method shows an efficient way to incorporate inverse dynamics to engineering multibody applications, e.g., trajectory tracking or parameter identification in the field of robotics. The presented method is applicable for examples for both, parameter identification and optimal control, and shows a high potential for broad applicability for a wide range of optimization problems in multibody dynamics. Especially, in case of parameter identifications in engineering multibody applications, a theoretical enhancement of the proposed adjoint method by an error control of accelerations is inevitable in order to meet the circumstances of experimental studies using acceleration sensors in general.

1 ADJOINT METHOD

The adjoint method is well suited to solve a variety of optimization problems in engineering sciences. Much attention to this approach has been paid recently in the context of continuous systems, described by partial differential equations, see e.g. [6]. However, in multibody dynamics the adjoint method is hardly ever applied, since the structure of the equations of motion is usually extremely complicated, in particular if flexible bodies are included. The effort to obtain the set of adjoint equations seems to be tremendously high and therefore the adjoint method is obviously unattractive for most developers of multibody simulation software. In this paper we discuss the practical applicability and the high potential of the adjoint method for classical optimization problems in multibody dynamics such as inverse dynamics and parameter identification.

The basic idea of the adjoint method, see e.g. [1] or [4], may be described as follows. Suppose that we have the semi-explicit differential algebraic system

$$\dot{x} = f(x,\lambda,u,t),\; x(0) = x_0,\;\; c(x) = 0 \tag{1}$$

where $x(t)$ and $\lambda(t)$ are the vectors of state variables and algebraic variables, as it arises in redundant formulations of multibody system dynamics. Herein, u may either describe a vector of constant parameters or a vector of control signals. Our goal is to find u such that the measured signals $\bar{s}_i(t), i=1...n$ are best approximated by the system outputs $s_i(x)$, i.e. we are looking for the minimum of the following cost functional

$$J(u) = \int_0^T \sum_{i=1}^{n} \left[s_i(x) - \bar{s}_i(t)\right]^2 dt = \int_0^T g(x,t)\, dt \tag{2}$$

To minimize the functional $J(u)$ one can apply the classical method of the steepest descent, the conjugate gradient method, the Gauss-Newton method or quasi Newton methods like the BFGS algorithm. Some authors embed these methods in a homotopy continuation to obtain a global minimum, see e.g. [7]. In any cases the goal is to find the gradient of $J(u)$ in an efficient way. For this purpose several strategies can be pursued. On the one hand, if u is a vector of *m* parameters, the sensitivity equations for $x_u = \partial x / \partial u$ can be considered, see e.g. [3] or [7]. The computational effort for this approach is equal to solving *m* linear sets of equations with the same dimension as Eq. (1). On the other hand, in case $u(t)$ is a vector of control signals, which are usually discretized, the problem could be transformed to a finite dimensional one.

Here, the adjoint method turns out to be a powerful alternative to compute the gradient of $J(u)$ in both cases. As a first step, the functional $J(u)$ is augmented by adding Eq. (1) to the integrand - while not changing the result – following to

$$J(u) = \int_0^T \left[g + y^{\mathrm{T}} (f - \dot{x}) + \mu^{\mathrm{T}} c \right] dt \tag{3}$$

choosing arbitrary functions $y(t)$ and $\mu(t)$. The Hamiltonian function is introduced as

$$H(x, y, \lambda, \mu, u, t) = g(x,t) + y^{\mathrm{T}} f(x, \lambda, u, t) + \mu^{\mathrm{T}} c(x) \tag{4}$$

which is used to reformulate Eq. (3) as

$$J(u) = \int_0^T \left[H - y^{\mathrm{T}} \dot{x} \right] dt \tag{5}$$

The gradient of the functional $J(u)$ is obtained by integrating by parts of Eq. (5) leading to

$$\begin{aligned} \nabla J(u) &= \int_0^T \left[H_u + H_x^{\mathrm{T}} x_u + H_\lambda^{\mathrm{T}} \lambda_u - y^{\mathrm{T}} \dot{x}_u \right] dt \\ &= \int_0^T \left[H_u + (H_x^{\mathrm{T}} + \dot{y}^{\mathrm{T}}) x_u + H_\lambda^{\mathrm{T}} \lambda_u \right] dt - y^{\mathrm{T}} x_u \Big|_0^T \end{aligned} \tag{6}$$

The computation of the sensitivities x_u and λ_u can be avoided if the adjoint variables $y(t)$ and $\mu(t)$ are chosen such that

$$\dot{y} = -H_x, \quad y(T) = 0 \text{ and } H_\lambda = 0 \tag{7}$$

This set of semi-explicit differential algebraic equations in Eq. (7) is called the adjoint system of Eq. (1). The adjoint system is solved backwards in time starting at $t = T$, once the original equations have been solved forward. The gradient of $J(u)$ is immediately given by

$$\nabla J(u) = \int_0^T H_u \, dt \tag{8}$$

with $x(t), \lambda(t), y(t)$ and $\mu(t)$ computed from Eqs. (1) and (7). If u is a control signal, the variation of the functional $J(u)$ reads

$$\delta J(u) = \int_0^T H_u^{\mathrm{T}} \, \delta u \, dt \tag{9}$$

and the direction of the steepest ascent is given by $\delta u(t) = H_u$, see also [6]. It has to be emphasized here, that only two systems of DAEs must be integrated for that purpose. By solving the *m* sensitivity equations and *m* second order adjoint systems it is even possible to compute the Hesse matrix of $J(u)$ in a similar way, see e.g. [2] or [5].

2 ADJOINT METHOD IN MULTIBODY DYNAMICS

The main difficulty when applying the adjoint method in multibody dynamics results from the complexity of the original system in Eq. (1). Hence, many authors focused on two-dimensional examples or rather general aspects, as e.g. [3]. However, based on highly redundant formulations the adjoint equations in Eq. (7) for a multibody system are relatively simple. A mechanical multibody system consisting of rigid and flexible bodies, forces and constraints acting on the system can be described by the equations of motion

$$
\begin{aligned}
&M(q)\ddot{q} = f(q,\dot{q},u) - C_q^{\,T}(q)\lambda \\
&C(q) = 0
\end{aligned}
\tag{10}
$$

in which q denotes the vector of redundant generalized coordinates. The constraint equation $C(q) = 0$ enters the equation of motion via the constraint Jacobian C_q and in combination with the vector of Lagrange multiplier λ the constraint forces acting on the system are described. The system may include a vector of parameters u or a control $u(t)$ in addition. It has to be mentioned here that only the cases are discussed in which u appears as stiffness or damping parameter or as actuating force. The method may be extended to the case in which u appears in the constraint equation too.

The system is reformulated to a first order system of equations by using additional variables $v = \dot{q}$ as follows

$$
\begin{aligned}
&\dot{q} = v \\
&M\dot{v} = f(q,v,u) - C_q^{\,T}\lambda \\
&C(q) = 0
\end{aligned}
\tag{11}
$$

The goal is to determine the parameter or control vector which minimizes the functional

$$
J(u) = \int_0^T h(q,v,\dot{v},u)dt + S(q,v)\big|_{t=T} \tag{12}
$$

in which the state vector x from Eq. (2) is now represented by q and v. An additional end point term for describing an end point root-mean-square-error of a sensor variable can be included as a scrap function here, defined by $S(q,v)\big|_{t=T}$. As a first step, similar to the derivation of Eq. (3), the functional $J(u)$ in Eq. (12) is augmented by adding the equations of motion of a multibody system in Eq. (11) leading to

$$
J = \int_0^T \left(h + p^T(\dot{q} - v) + w^T(M\dot{v} - f + C_q^{\,T}\lambda) + \mu^T C\right)dt + S\big|_{t=T} \tag{13}
$$

written without variable dependencies for a better readability. Here, the so called adjoint variables $p(t), w(t)$ and $\mu(t)$ may be chosen arbitrarily. Following the idea presented above, the variation of the functional in Eq. (13) and the ensuing integration by parts for the terms including $\delta\dot{q}$ and $\delta\dot{v}$ is performed. Collecting the terms multiplied by $\delta q, \delta v, \delta\lambda$ and δu and equating the respective expressions to zero leads to the following adjoint system of equations

$$
\begin{aligned}
&\frac{dp}{dt} = h_q^{\,T} + Aw + C_q^{\,T}\mu \\
&\frac{d}{dt}(Mw) = h_v^{\,T} - p - Bw - \frac{d}{dt}(h_{\dot{v}}^{\,T}) \\
&0 = C_q w \\
&0 = S_q^{\,T} + p(T) \\
&0 = S_v^{\,T} + M(T)w(T) + h_{\dot{v}}^{\,T}
\end{aligned}
\tag{14}
$$

with $A = (M\dot{v})_q{}^T - f_q{}^T + (C_q{}^T \lambda)_q{}^T$ and $B = f_v{}^T$ and the symmetry of the mass matrix $M = M^T$ has been used. The gradient of the corresponding functional reduces to

$$\delta J = \int_0^T (h_u - w^T f_u)\, \delta u \, dt \qquad (15)$$

which directly relates the variation δu to the variation of the objective function. It has to be mentioned that the boundary condition for the adjoint variable w in Eq. (14.V) is generally incompatible with the constraint equation in Eq. (14.III) at $t = T$. Therefore, consistent boundary conditions for the adjoint system have to be defined, as e.g. proposed in [8].

As well in [8], a backward differentiation scheme for the adjoint system is defined and some remarks on the computation of Jacobian matrices A and B in Eq. (14) may help to improve the performance of a parameter identification or optimal control problem.

3 CONCLUSIONS

The proposed method shows the embedding of the adjoint method in multibody system dynamics and its potential for parameter identification and optimal control on account of its linear structure. Despite the complex structure of the original multibody system, the adjoint method presents an elegant and efficient way to incorporate inverse dynamics to flexible multibody system applications arising from modern engineering problems. The enhanced adjoint method includes the error of accelerations in the optimization functional and therefore delivers an elegant and efficient way to incorporate inverse dynamics for parameter identification in flexible multibody system applications arising from modern engineering problems including acceleration sensors.

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