Ball On A Plate – Control System

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ABSTRACT:
In this work an autonomous system for balancing and orbit control of a metal ball on a moving flat plate was developed, designed and successfully realized. The position detection of the ball is realized using a resistive touchpad on a glass plate. The panel can be tilted by two digital servo motors via a central universal joint in two solid angles. Both, position detection of the ball and motor drive require a control algorithm which was implemented on a microcontroller board. The mathematical modeling of the dynamic system behavior consisting of the servo drive, mechanics of ball and structure plus the touchpad have been verified by measurements. Based on this analytical model a classical PID controller as well as a modern state space control including state observer have been realized. Thus fast and accurate control as well as disturbance responses for PID/state control showing a settling time of tset=6s/2s with an overshoot of □h=25%/0% respectively have been achieved. Since the constructed system is built to be used for laboratory exercises, even further more advanced algorithms like Linear-Quadratic Regulator (LQR), Dead-Beat, Fuzzy, etc. can be implemented and compared easily.

Keywords: Ball On A Plate, modelling, PID control, state space, observer, microcontroller.

1 INTRODUCTION

The design of the system was intended to be built in a clean and durable way for universal use for laboratory exercises as part of the control theory course. Therefore a comprehensive state of the art research of already realized ball on a plate systems was done. First the position detection of the ball, representing a key element of the system, was decided to be realized with a nearly perfect flat glass plate resistive touchpad featuring easy readout instead of bulky camera based systems used in [1, 2]. Though the realization in [2] uses linear

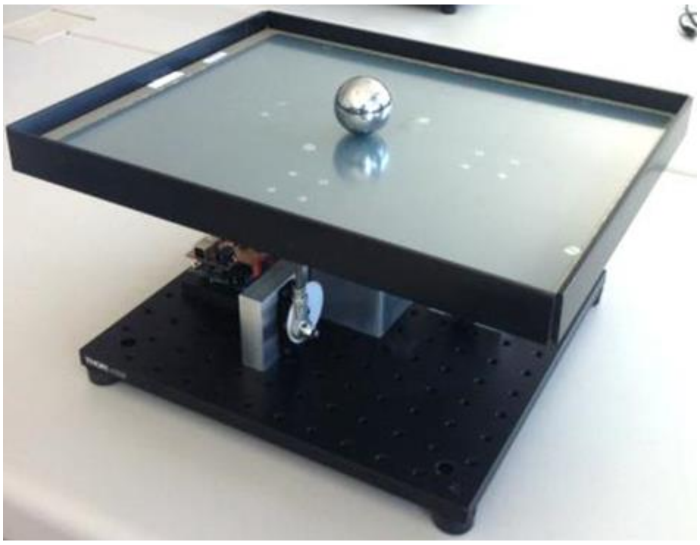

Figure 1. Realized ball on a plate system.

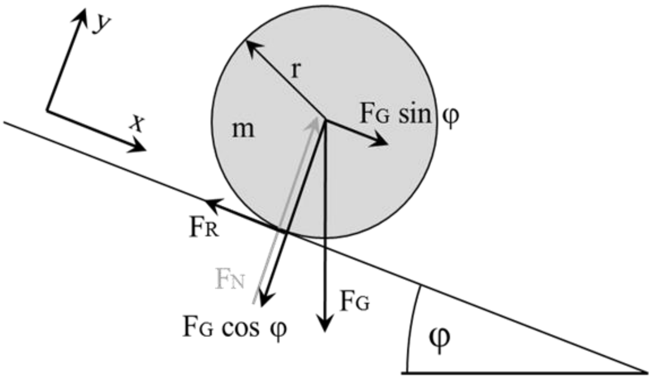

Figure 2. Geometric representation of the forces.

moving magnetic actuators, a compact implementation with two digital servo motors, one for each angle, with rods for the plate motion, is preferred. In [3] a complex mechanical construction featuring outer joints is shown. Here a simple central joint has been designed to carry the plate and ensure proper independent movement of both angles. The mechanical system was completely constructed via 3D CAD model. Thus the required mechanical parts could be manufactured easily. Fig. 1 shows the realized ball on a plate system. The system features a microcontroller board type STM32F4-Discovery-Board [4] for readout of the ball position from the touchpad and real-time calculation of the required control signal for the two decoupled control loops with a servo motor each. The functional setup is first used for system identification and then compared to the analytical model. Out of this the controller synthesis of a classical PID controller as well as state space control including state observer is shown next. The TUSTIN discretization [5] of these control algorithms permit real-time implementation on the microcontroller. Next the step responses for the two controllers are being measured and compared. Finally this setup is open for the implementation and comparison of other advanced control algorithms.

2 MODELLING OF THE SYSTEM

Prior to the synthesis of the controller the dynamic behavior of the system has to be identified. The system of a single axis contains the servo motor, the mechanical part and the touchpad to be used as a position sensor. For the dynamic behavior of the servo motor with input *ΔPWM(s)* and output angle *Φ(s)* a PT_1 transfer element $G_{servo}(s)$ with a gain $k_{servo}=0.03[rad/\%]$ and a time constant T_1=32ms have been measured.

$$G_{servo}(s) = \frac{\Phi(s)}{\Delta \mathrm{PWM}(s)} = \frac{k_{servo}}{1+sT_1} \qquad (1)$$

The mechanical part can be determined analytically by a short analysis of the forces (fig. 2). Therefore the following equations result together with the parameters: mass *m* and radius *r* of the ball, acceleration *a*, gravity acceleration *g*, angle of inclination φ, angular acceleration α=a/r, torque of inertia $J=0.4mr^2$ of the ball [6] and finally normal force F_N and frictional force F_R which can be neglected for the motion of the steel ball on a glass plate.

$$m\,a = m\,g\,sin\,\varphi - F_R \qquad \text{x-direction} \qquad (2)$$

$$0 = F_N - m\,g\cos\varphi \qquad \text{y-direction} \qquad (3)$$

$$F_R\,r = J\,\alpha \qquad \text{rotation} \qquad (4)$$

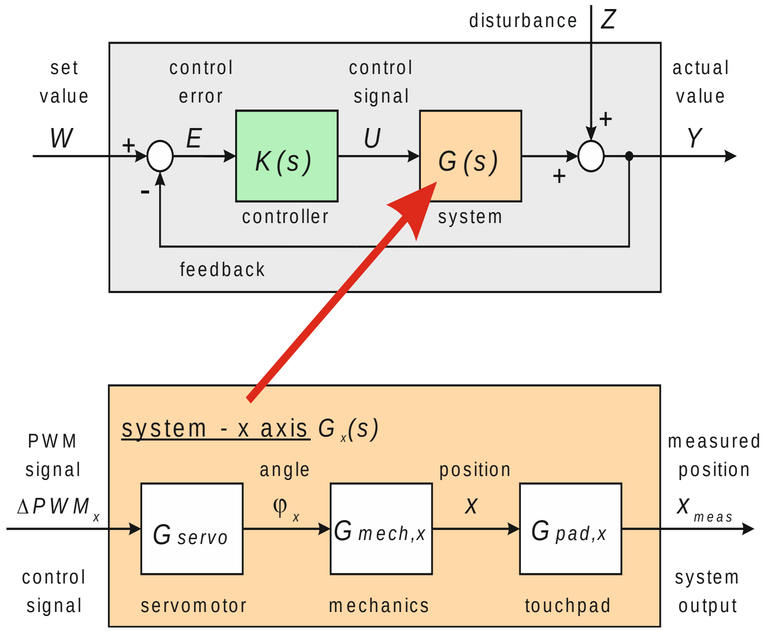

Figure 3. a.) Standard control loop and b.) block diagram of the system.

Together with the legitimate linearization for small angles $\sin\varphi \approx \varphi$, the linear acceleration and the position of the ball result in the following step.

$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{g\ \sin\varphi}{1.4} \approx \frac{g\ \varphi}{1.4} \tag{5}$$

$$x(t) = \int\int a(t)\ dt^2 = \frac{g\ \varphi}{1.4}\ \frac{t^2}{2} + t\ v_0 + x_0 \tag{6}$$

Next the LAPLACE transformation is applied to (6) for neglected initial conditions and then divided by an assumed angular input step. This leads directly to the searched transfer function of the mechanical system G_{mech}(s) with input angle *Φ(s)* and output position *X(s)* and I_2-element properties with a gain of k_{mech}=7[m/rad].

$$G_{mech}(s) = \frac{X(s)}{\Phi(s)} = \frac{g}{1.4\ s^2} = \frac{k_{mech}}{s^2} \tag{7}$$

The touchpad measures the ball position with a resolution of 12bit for a maximal x-distance of 0.34m, ending up in the P-transfer function $G_{pad,x}$(s) with input position *X(s),* output measurement value $X_{meas}(s)$ and gain $k_{pad,x}$=1.2·10^4[px/m].

$$G_{pad,x} = \frac{X_{meas}(s)}{X(s)} = \frac{2^{12}\ [px]}{0.34\ [m]} = k_{pad,x} \tag{8}$$

Out of these transfer elements the complete I_2T_1-system transfer function for x-direction $G_x(s)$ results with a system gain $k_s = k_{servo}\ g\,\frac{k_{pad}}{1.4} = 2532\left[\frac{pix}{\%}\right]$, see fig. 3 b.).

$$G_x(s) = \frac{X(s)}{\Delta PWM_x(s)} = G_{servo}\ G_{mech}\ G_{pad} = \frac{k_s}{s^2(1+sT_1)} \tag{9}$$

$$G_y(s) = \frac{Y(s)}{\Delta\mathrm{PWM}_y(s)} = \frac{3}{4}\, G_x(s) \tag{10}$$

Both x- and y-directions can be considered to be decoupled due to the chosen centered arrangement of the servos under the plate. Therefore only the touchpad aspect ratio of 4:3 has to be considered for the complete system transfer function in y-direction $G_y(s)$. Fig. 4 shows a good match in the comparison of the measured and the modelled step response of the x-axis for a maximum angular step of Δφ=+17°.

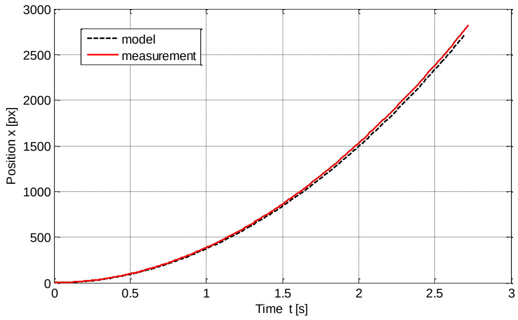

Figure 4. Comparison of measured and modelled system step response.

3 CONTROLLER SYNTHESIS

Based on the derived transfer function of the I_2T_1-system, an appropriate controller can now be designed. First a simple $PIDT_1$ controller is developed followed by the design of a more complex state control requiring a state observer.

3.1 Standard control loop

Looking at the bode plot of the I_2T_1-system with a phase of $\varphi_G \leq -180°$ (see fig. 5), it becomes clear that a controller with derivative (D) behavior for phase lead is required due to stability reasons in order to obtain a sufficient phase margin. Further integral (I) controller behavior is desired for accurate control. Therefore an appropriate $PIDT_1$ controller for the corresponding transfer function *K(s)* with the parameters k_p=0.004, k_i=0.003, k_d=0.025 and T_x=10ms can be directly designed in the bode-plot.

$$K(s) = k_p + \frac{k_i}{s} + \frac{sk_d}{1+sT_x} \tag{11}$$

Together with the open loop transfer function $F_o(s)$, the control transfer function *T(s)* of the standard control loop results according to fig. 3a.) and [7,8]

$$T(s) = \frac{Y(s)}{W(s)} = \frac{F_o(s)}{1+F_o(s)} \; and \;\; F_o(s) = K(s)G(s) \tag{12}$$

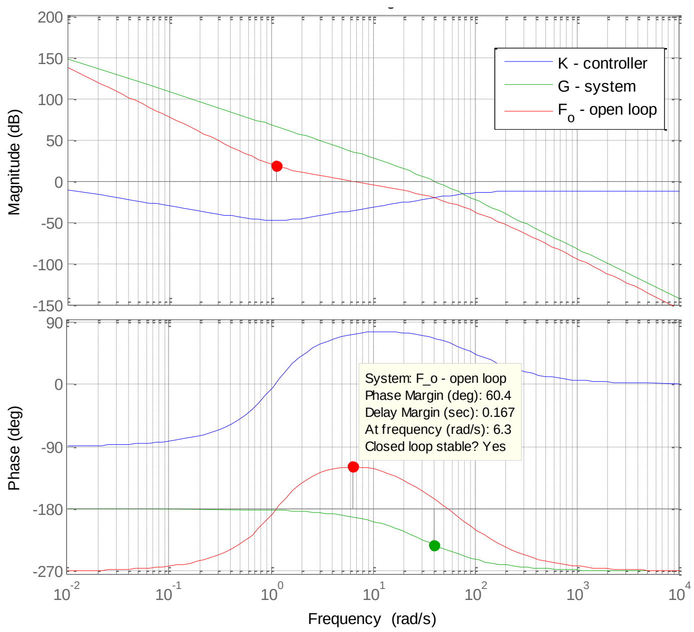

Figure 5. Bode plot of system *G(s)*, controller *K(s)* and open loop $F_o(s)$.

Moreover the disturbance transfer function $F_z(s)$ is given.

$$F_z(s) = \frac{y(s)}{z_x(s)} = \frac{1}{1+F_o(s)} \tag{13}$$

Fig. 5. shows the bode plots of the system *G(s)*, controller *K(s)* and open loop $F_o(s)$. Here a phase margin α_R=60° at crossover frequency ω_D=6.3rad/s is achieved. Thus promising stable and accurate control of the ball on a plate system can be expected using this standard control loop.

3.2 State control

A modern approach for the control of the ball on a plate problem uses the state space description of the system [9].

$$\dot{x}(t) = \boldsymbol{A}\,x(t) + \boldsymbol{B}\,u(t) \tag{14}$$

$$y(t) = \boldsymbol{C}\,x(t) + \boldsymbol{D}\,u(t) \tag{15}$$

Therefore the transfer function of the system model of 3rd order without feedthrough can directly be transferred to the state space representation, where we use the controller canonical form [10] according to (9, 10).

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/T_1 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{16}$$

$$\boldsymbol{C} = [k_s/T_1 \quad 0 \quad 0] \qquad \boldsymbol{D} = [0] \tag{17}$$

3.3 Pole placing

Compared to the indirect PID controller synthesis via phase margin of the open loop transfer function, the control response of the state space control can directly be defined via pole placing of the chosen stable poles $P=\{p_1, p_2, p_3\}=\{-3.5+3.5j, -3.5-3.5j, -10.5\}$. Thus the state controller $\boldsymbol{K}$ is determined. Furthermore a prefilter *V* is required to ensure accurate control response. The complete state control for one axis is shown in fig. 6, see also [5,7,9,10].

$$det(s\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{BK}) = (s-p_1)(s-p_2)(s-p_3) \tag{18}$$

$$\boldsymbol{K} = [k_1 \quad k_2 \quad k_3] = [-257.25 \quad -98 \quad 13.75] \tag{19}$$

$$V = -[\boldsymbol{C}(\boldsymbol{A}+\boldsymbol{BK})^{-1}\boldsymbol{B}]^{-1} = 0.0035 \tag{20}$$

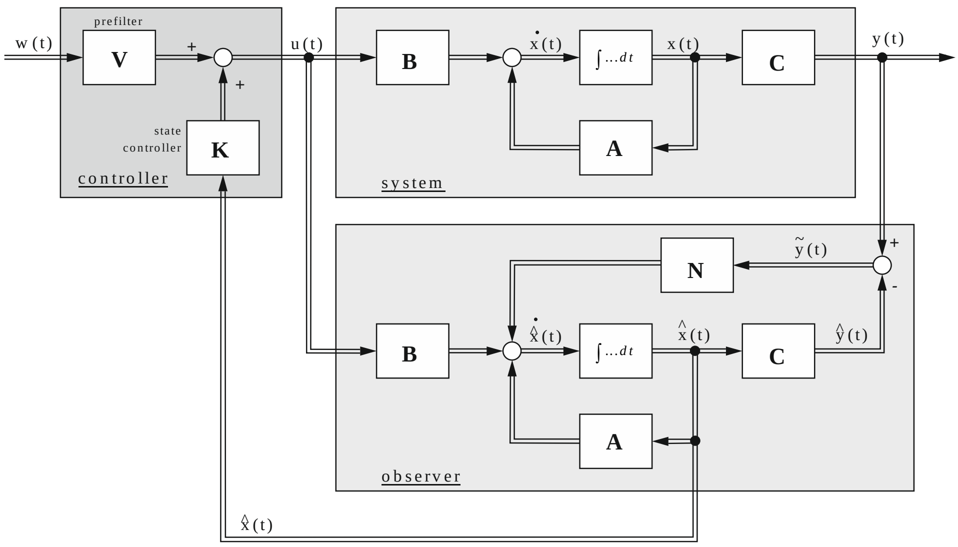

Figure 6. Single axis state control.

3.4 State observer

The state space controller requires the input of the complete state variable, which cannot directly be measured. Therefore the actual states of the system are calculated in a parallel model, the state observer. Comparing the actual and the estimated output $\Delta y(t) = y(t) - \hat{y}(t)$, the estimated state variable is accurate if the dynamic of the observer is higher than of the system. The design of the observer ***N*** is done similar to the controller design via pole placement using $Q=\{q_1, q_2, q_3\}=\{-18\ -18\ -80\}$. Both, controllability and observability of the system has been proven. Further separate synthesis of controller and observer is mentioned.

$$det(s\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{NC}) = (s - q_1)(s - q_2)(s - q_3) \tag{21}$$

$$\boldsymbol{N} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} 0.0012 \\ 0.0075 \\ 0.1162 \end{bmatrix} \tag{22}$$

3.5 Discretization

For the implementation of the PIDT1 control algorithms a discretization with the sample time of the microcontroller board T=10ms is done. Using the Tustin approximation [8] we switch from continuous Laplace to discrete Z-transform of the controller transfer function $K(s) = \frac{U(s)}{E(s)} \rightarrow K(z) = \frac{U(z)}{E(z)}$ by substituting $s = \frac{2}{T}\frac{z+1}{z-1}$. Thus the following control algorithm with continuous cycle count k is obtained and can directly be implemented into the microcontroller board. For the discretization of the state control see [5].

$$u_k = 1.98u_{k-1} - 0.98u_{k-2} + 0.17e_k - 0.23e_{k-1} + 0.06e_{k-2} \tag{23}$$

4 RESULTS

A settling time of t_{set}~6s and an overshoot of Δh=25% was achieved for the command response using the PID controller. This is a good behavior for the simple standard control loop due to the fact that the controller can react only to the complete delayed system response. Moreover the state feedback control shows a step response with t_{set}=2s and Δh=0%. This significant improvement compared to the standard control results from the more direct feedback of the state space variables. Fig. 7 shows a comparison of both measured step responses. Also examined disturbance response of the realized controllers is quite satisfactory when deflecting the ball manually. Table 1 gives a comparison of control and disturbance responses for the realized system and similar realizations. The results of the other systems have been significantly exceeded. A further improvement of the dynamic behavior would require faster servo actuators and is not necessary for our demands.

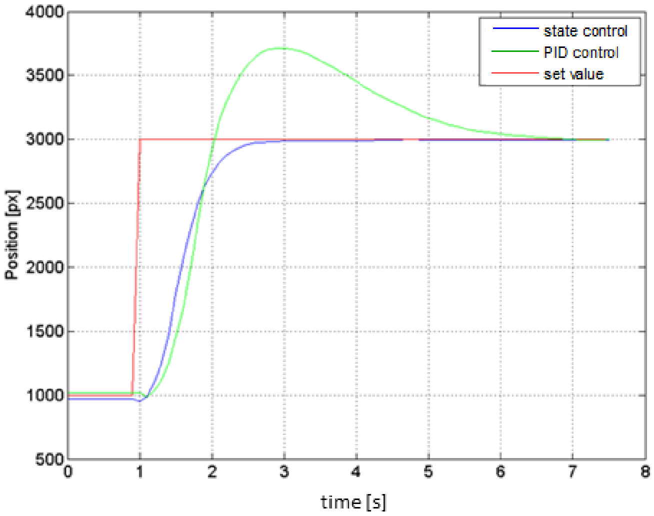

Figure 7. Measured step responses: a.) PID and b.) state-space control.

Table 1. Comparison of measured command and disturbance step response to state of the art results.

		this work		[1]	[2]	[3]	[11]	[12]
		PID	State	Fuzzy	PID	Back	State	State
command response	t_{set}	≤ 6s	≤ 2s	8s	-	3s	≥ 2s	8s*
	Δh	25%	0%	0%	-	2%	-	20%*
disturbance response	t_{set}	≤ 6s	≤ 2s	-	-	5s	≥ 2s	
	Δh	25%	0%	-	-	3%	-	

* reported response is not accurate in steady state

5 CONCLUSION

The stabilization of the realized ball on a plate system has been successfully shown using a PID control and a state control respectively. First a model of the complete system has been established and could be confirmed by measurements of the system step response. Next a standard control loop with sufficient phase margin has been designed. The implementation of the advanced state control led to improved command and disturbance responses. Further control algorithms like LQR, Dead-Beat Fuzzy, or non-linear controller can now easily be implemented and compared.

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